# Model Reference Gain Scheduling Control of a PEM Fuel Cell using Takagi-Sugeno Modelling

Damiano Rotondo, Vicenç Puig, and Fatiha Nejjari

Advanced Control Systems Group (SAC), Universitat Politècnica de Catalunya (UPC), TR11, Rambla de Sant Nebridi, 10, 08222 Terrassa, Spain.

**Abstract.** In this paper, a solution for the oxygen stoichiometry control problem for Proton Exchange Membrane (PEM) fuel cells is presented. The solution relies on the use of a reference model, where the resulting nonlinear error model is brought to a Takagi-Sugeno (TS) form using the non-linear sector approach. The TS model is suitable for designing a controller using Linear Matrix Inequalities (LMI)-based techniques, such that the resulting closed-loop error system is stable with poles placed in some desired region of the complex plane. Simulation results are used to show the effectiveness of the proposed approach. In particular, the PEM fuel cell can reach asymptotically the oxygen stoichiometry set-point despite all the considered stack current changes.

**Keywords:** Takagi-Sugeno model, Reference model based control, Gain-scheduling, PEM Fuel Cell, LMIs

## 1 Introduction

Proton Exchange Membrane (PEM, also known as Polymer Electrolyte Membrane) fuel cells are one of the most promising technologies to be used, in a near future, as power supply sources in many portable applications. A good performance of these devices is closely related to the kind of control that is used, so a study of different control alternatives is justified [1]. A fuel cell integrates many components into a power system, which supplies electricity to an electric load or to the grid. Several devices, such as DC/DC or DC/AC converters, batteries or ultracapacitors, are included in the system and, in case the fuel cell is not fed directly with hydrogen, a reformer must also be used. Therefore, there are many control loops schemes depending on the devices that must be controlled. The lower control level takes care of the main control loops inside the fuel cell, which are basically fuel/air feeding, humidity, pressure and temperature. The upper control level is in charge of the whole system, integrating the electrical conditioning, storage and reformer (if necessary). Many control strategies have been proposed in literature, ranging from feedforward control [1], LQR [2] or Model Predictive Control [3].

Recently, the complex and non-linear dynamics of the power generation systems based on fuel cell technology, described in detail in [4], led to the use of linear models that include parameter varying with the operating point (known as LPV models) not only for advanced control techniques [5] but also for model-based fault diagnosis algorithms [6]. The use of Takagi-Sugeno (TS) models [7] is an alternative to the LPV models, as proposed in [8]. This paper will follow this last approach.

In this paper, a solution for the oxygen stoichiometry control problem for PEM fuel cells using TS models is presented. The solution relies on the use of a reference model, where the resulting nonlinear error model is brought to a TS form using the non-linear sector approach [9]. The TS model is suitable for designing a controller using Linear Matrix Inequalities (LMI)-based techniques, such that the resulting closed-loop error system is stable with poles placed in some desired region of the complex plane [10]. Simulation results are used to show the effectiveness of the proposed approach. In particular, the PEM fuel cell can reach asymptotically the oxygen stoichiometry set-point despite all the considered stack current changes.

The structure of the paper is the following: Section 2 shows how, starting from the non-linear model of a PEM fuel cell, and using a reference model, a model of the error dynamics suitable for TS modelling can be derived. Section 3 presents the methodology to design a TS controller based on the TS model of the error. Section 4 illustrates the performance of the proposed TS control strategy in simulation. Finally, Section 5 provides the main conclusions and future work.

## 2 Model Reference Control of the PEM Fuel Cell System

### 2.1 PEM Fuel Cell Description

A fuel cell is an electrochemical energy converter that converts the chemical energy of fuel into electrical current. It has an electrolyte, a negative electrode and a positive electrode, and it generates direct electrical current through an electrochemical reaction. Typical reactants for fuel cells are hydrogen as fuel and oxygen as oxidant that, once the reaction takes place, produce water and waste heat.

The basic physical structure of a fuel cell consists of an electrolyte layer in contact with a porous anode and cathode electrode plates. There are different kinds of electrolyte layers. Here a PEM (Polymer Electrolyte Membrane or Proton Exchange Membrane) fuel cell is used. The PEM has a special property: it conducts protons but is impermeable to gas (the electrons are blocked through the membrane). Auxiliary devices are required to ensure the proper operation of the fuel cell stack.

### 2.2 PEM Fuel Cell System Model

The model used in this work has been presented in [4]. The model is widely accepted in the control community as a good representation of the behaviour of a Fuel Cell Stack (FCS) system.

**Air Compressor.** The air compressor is decomposed into two main parts. One part concerns the electric motor, whereas the other part concerns the compressor box. The compressor motor is modelled using a direct current electric motor model. A compressor flow map is used to determine the air flow rate $W_{cp}$, supplied by the compressor. The model of the air compressor is given by:

$$\dot{\omega}_{cp} = \frac{\eta_{cp}}{J_{cp}} \frac{k_t}{R_{cm}} \left( \upsilon_{cm} - k_v \omega_{cp} \right) - \frac{C_p T_{atm}}{J_{cp} \omega_{cp} \eta_{cp}} \left[ \left( \frac{p_{sm}}{p_{atm}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] W_{cp} \tag{1}$$

where $\upsilon_{cm}$ is the motor supply voltage (V).

**Supply Manifold.** Manifolds are modelled as a lumped volume in pipes or connections between different devices. The following differential equation is used to model the supply manifold pressure behaviour:

$$\dot{p}_{sm} = \frac{\gamma R_a}{V_{sm}} \left\{ W_{cp} \left[ T_{atm} + \frac{T_{atm}}{\eta_{cp}} \left[ \left( \frac{p_{sm}}{p_{atm}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right] - k_{sm,out} \left( p_{sm} - \frac{m_{O_2,ca} R_{O_2} T_{st}}{V_{ca}} \right) T_{sm} \right\} \quad (2)$$

**Return Manifold.** An equation similar to the one introduced for the supply manifold is used to describe the return manifold behaviour:

$$\dot{p}_{rm} = \frac{R_a T_{rm}}{V_{rm}} \left[ k_{ca,out} \left( \frac{m_{O_2,ca} R_{O_2} T_{st}}{V_{ca}} - p_{rm} \right) - k_{rm,out} \left( p_{rm} - p_{atm} \right) \right] \quad (3)$$

**Anode Flow Dynamics.** The hydrogen supplied to the anode is regulated by a proportional controller. The controller takes the differential pressure between anode and cathode to compute the regulated hydrogen flow:

$$\dot{m}_{H_2,an} = K_1 (K_2 p_{sm} - \frac{m_{H_2,an} R_{H_2} T_{st}}{V_{an}}) - M_{H_2} \frac{n I_{st}}{2F} \quad (4)$$

**Cathode Flow Dynamics.** The cathode flow dynamics is described by the following differential equation:

$$\dot{m}_{O_2,ca} = k_{sm,out} p_{sm} - \frac{m_{O_2,ca} R_{O_2} T_{st}}{V_{ca}} (k_{sm,out} + k_{ca,out}) + k_{ca,out} p_{rm} - M_{O_2} \frac{n I_{st}}{4F} \quad (5)$$

**Oxygen Stoichiometry.** The efficiency optimization of the current system can be achieved by regulating the oxygen mass inflow toward the stack cathode [11]. If an adequate oxidant flow is ensured through the stack, the load demand is satisfied with minimum fuel consumption. In addition, oxygen starvation and irreversible damage are averted. To accomplish such an oxidant flow is equivalent to maintaining at a suitable value the oxygen stoichiometry, defined as:

$$\lambda_{O_2} = \frac{k_{sm,out} \left( p_{sm} - \frac{m_{O_2,ca} R_{O_2} T_{st}}{V_{ca}} \right)}{M_{O_2} \frac{n I_{st}}{4F}} \quad (6)$$

The overall model has five state variables (the compressor speed $\omega_{cp}$ (rad/s), the pressure in the supply manifold $p_{sm}$ (Pa), the pressure in the return manifold $p_{rm}$ (Pa), the mass of hydrogen in the anode $m_{H_2,an}$ (kg) and the mass of oxygen in the cathode $m_{O_2,ca}$ (kg)) and three inputs, two of which can be used as control variables (the compressor mass flow $W_{cp}$ (kg/s) and the return manifold outlet orifice constant $k_{rm,out}$ (ms)) while the other (the current in the stack $I_{st}$ (A)) can be considered as a disturbance input that can be included in the reference model in order to generate an appropriate feedforward action and make the feedback loop insensitive to its variations. The values used in this work have been taken from [12], and are listed in Table 1.

### 2.3 Reference Model

Let us define the following reference model:

$$\dot{p}_{sm}^{ref} = \frac{\gamma R_a}{V_{sm}}\left\{W_{cp}^{ref}\left[T_{atm} + \frac{T_{atm}}{\eta_{cp}}\left[\left(\frac{p_{sm}}{p_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right] - k_{sm,out}\left(p_{sm}^{ref} - \frac{m_{O_2,ca}^{ref} R_{O_2} T_{st}}{V_{ca}}\right)T_{sm}\right\} \tag{7}$$

$$\dot{p}_{rm}^{ref} = \frac{R_a T_{rm}}{V_{rm}}\left[k_{ca,out}\left(\frac{m_{O_2,ca}^{ref} R_{O_2} T_{st}}{V_{ca}} - p_{rm}^{ref}\right) - k_{rm,out}^{ref}\left(p_{rm} - p_{atm}\right)\right] \tag{8}$$

$$\dot{m}_{O_2,ca}^{ref} = k_{sm,out} p_{sm}^{ref} - \frac{m_{O_2,ca}^{ref} R_{O_2} T_{st}}{V_{ca}}\left(k_{sm,out} + k_{ca,out}\right) + k_{ca,out} p_{rm}^{ref} - M_{O_2}\frac{nI_{st}}{4F} \tag{9}$$

The reference model provides the state trajectory to be tracked by the real PEM fuel cell, starting from the reference inputs $W_{cp}^{ref}$ and $k_{rm,out}^{ref}$. The values of the reference inputs to be fed to the reference model (feedforward actions) are obtained from steady-state considerations about the fuel cell system, so as to keep the supply manifold pressure and the oxygen stoichiometry at some desired values $p_{sm}^{\infty}$ and $\lambda_{O_2}^{ref}$.

### 2.4 Error Model

By subtracting the reference model equations (7)-(9) and the corresponding system equations (2), (3), (5), and by defining the tracking errors $e_1 \triangleq p_{sm}^{ref} - p_{sm}$, $e_2 \triangleq p_{rm}^{ref} - p_{rm}$, $e_3 \triangleq m_{O_2,ca}^{ref} - m_{O_2,ca}$, and the new inputs $u_1 \triangleq W_{cp}^{ref} - W_{cp}$, $u_2 \triangleq k_{rm,out}^{ref} - k_{rm,out}$, the error model for the PEM Fuel Cell can be brought to the following representation:

$$\dot{e}_1 = -\frac{\gamma R_a}{V_{sm}} k_{sm,out} T_{sm}\left(e_1 - \frac{R_{O_2} T_{st}}{V_{ca}} e_3\right) + b_{11}\left(p_{sm}\right) u_1 \tag{10}$$

$$\dot{e}_2 = -\frac{R_a T_{rm} k_{ca,out}}{V_{rm}}\left(e_2 - \frac{R_{O_2} T_{st}}{V_{ca}} e_3\right) + b_{22}\left(p_{rm}\right) u_2 \tag{11}$$

$$\dot{e}_3 = k_{sm,out} e_1 + k_{ca,out} e_2 - \left(k_{sm,out} + k_{ca,out}\right)\frac{R_{O_2} T_{st}}{V_{ca}} e_3 \tag{12}$$

with:

$$b_{11}\left(p_{sm}\right) = \frac{\gamma R_a}{V_{sm}}\left[T_{atm} + \frac{T_{atm}}{\eta_{cp}}\left[\left(\frac{p_{sm}}{p_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right] \tag{13}$$

$$b_{22}\left(p_{rm}\right) = -\frac{R_a T_{rm}}{V_{rm}}\left(p_{rm} - p_{atm}\right) \tag{14}$$

| Variable | Description | Value and Unit |
|---|---|---|
| $\eta_{cp}$ | Compressor efficiency | 0.8 |
| $\gamma$ | Specific heat capacity of gas | 1.4 |
| $R_a$ | Air gas constant | $286.9\,J/(kgK)$ |
| $R_{O_2}$ | Oxygen gas constant | $259.8\,J/(kgK)$ |
| $R_{H_2}$ | Hydrogen gas constant | $4124.3\,J/(kgK)$ |
| $V_{sm}$ | Supply manifold volume | $0.02\,m^3$ |
| $V_{ca}$ | Cathode volume | $0.01\,m^3$ |
| $V_{rm}$ | Return manifold volume | $0.005\,m^3$ |
| $V_{an}$ | Anode volume | $0.005\,m^3$ |
| $T_{atm}$ | Air temperature | $298.15\,K$ |
| $T_{st}$ | Temperature in the stack | $350\,K$ |
| $T_{sm}$ | Supply manifold temperature | $300\,K$ |
| $T_{rm}$ | Return manifold temperature | $300\,K$ |
| $p_{atm}$ | Air pressure | $101325\,Pa$ |
| $k_{sm,out}$ | Supply manifold outlet flow constant | $0.3629 \cdot 10^{-5}\,kg/sPa$ |
| $k_{ca,out}$ | Cathode outlet flow constant | $0.2177 \cdot 10^{-5}\,kg/sPa$ |
| $M_{H_2}$ | Hydrogen molar mass | $2.016 \cdot 10^{-3}\,kg/mol$ |
| $M_{O_2}$ | Oxygen molar mass | $32 \cdot 10^{-3}\,kg/mol$ |
| $n$ | Number of cells in the fuel cell stack | 381 |
| $F$ | Faraday constant | $96485\,C/mol$ |
| $K_1$ | Proportional gain | 2.1 |
| $K_2$ | Nominal pressure drop coefficient | 0.94 |
| $J_{cp}$ | Combined inertia of motor and compressor | $5 \cdot 10^{-5}\,kgm^2$ |
| $k_t$ | Torque constant | $0.0153\,Nm/A$ |
| $R_{cm}$ | Resistance | $0.82\,\Omega$ |
| $k_v$ | Motor constant | $0.0153\,Vs/rad$ |
| $C_p$ | Specific heat capacity of air | $1004\,J/kgK$ |

**Table 1.** List of parameters and values.

## 3 Controller Design Scheme

From the previous section, the PEM Fuel Cell error model can be expressed in a TS form, as follows:

$$\begin{array}{l} IF\ \vartheta_1(k)\ is\ M_{i1}\ AND\ \vartheta_2(k)\ is\ M_{i2} \\ THEN \begin{cases} e_i(k+1) = Ae(k) + B_i u(k) \\ y_i(k) = Ce(k) \end{cases} \qquad i = 1,\dots,N \end{array} \tag{15}$$

where $e \in \mathbb{R}^{n_e}$ is the error vector, $e_i \in \mathbb{R}^{n_e}$ is the error update due to the $i$-th rule of the fuzzy model, $u \in \mathbb{R}^{n_u}$ is the input vector, and $\vartheta_1(k)$, $\vartheta_2(k)$ are premise variables (in this paper, $\vartheta_1(k) = b_{11}(p_{sm}(k))$ and $\vartheta_2(k) = b_{22}(p_{rm}(k))$).

The entire fuzzy model of the error system is obtained by fuzzy blending of the consequent submodels. For a given pair of vectors $e(k)$ and $u(k)$, the final output of the fuzzy system is inferred as a weighted sum of the contributing submodels:

$$e(k+1) = \frac{\sum_{i=1}^{N} w_i\left(\vartheta(k)\right)\left[Ae(k)+B_iu(k)\right]}{\sum_{i=1}^{N} w_i\left(\vartheta(k)\right)} = \sum_{i=1}^{N} h_i\left(\vartheta(k)\right)\left[Ae(k)+B_iu(k)\right] \tag{16}$$

$$y(k) = Ce(k) \tag{17}$$

where $w_i\left(\vartheta(k)\right)$ and $h_i\left(\vartheta(k)\right)$ are defined as follows:

$$w_i\left(\vartheta(k)\right) = M_{i1}\left(\vartheta_1(k)\right)M_{i2}\left(\vartheta_2(k)\right) \qquad h_i\left(\vartheta(k)\right) = \frac{w_i(\vartheta(k))}{\sum_{i=1}^{N} w_i(\vartheta(k))} \tag{18}$$

where $M_{i1}\left(\vartheta_1(k)\right)$ and $M_{i2}\left(\vartheta_2(k)\right)$ are the grades of membership of $\vartheta_1(k)$ and $\vartheta_2(k)$ in $M_{i1}$ and $M_{i2}$, respectively, and $h_i\left(\vartheta(k)\right)$ is such that:

$$\sum_{i=1}^{N} h_i\left(\vartheta(k)\right) = 1 \quad h_i\left(\vartheta(k)\right) \geq 0 \quad i = 1,\dots,N \tag{19}$$

The error submodels in (15) are controlled through TS error-feedback control rules:

$$\begin{array}{l} IF\ \vartheta_1(k)\ is\ M_{i1}\ AND\ \vartheta_2(k)\ is\ M_{i2} \\ THEN\ u_i(k) = K_ie(k) \quad i = 1,\dots,N \end{array} \tag{20}$$

such that the overall controller output is inferred as the weighted mean:

$$u(k) = \sum_{i=1}^{N} h_i\left(\vartheta(k)\right)K_ie(k) \tag{21}$$

Since the vector of premise variables $\vartheta(k)$ is a function of the state variables $p_{sm}$ and $p_{rm}$, (21) represents a non-linear gain-scheduled control law. The goal of the controller design is to find the matrices $K_i$ such that the resulting closed-loop error system is stable with the poles of each subsystem in some desired region of the complex plane.

In this paper, both stability and pole clustering are analyzed within the quadratic Lyapunov framework, where a single quadratic Lyapunov function is used to assure the desired specifications. Despite the introduction of conservativeness with respect to other existing approaches, where the Lyapunov function is allowed to be parameter-varying, the quadratic approach has undeniable advantages in terms of computational complexity.

In particular, the TS error system (16), with the error-feedback control law (21), is quadratically stable if and only if there exist $X_s = X_s^T > 0$ and matrices $K_i$ such that [13]:

$$\begin{pmatrix} -X_s & \left(A+B_jK_i\right)X_s \\ X_s\left(A+B_jK_i\right)^T & -X_s \end{pmatrix} < 0 \qquad i,j = 1,\dots,N \tag{22}$$

On the other hand, pole clustering is based on the results obtained by [14], where subsets $\mathscr{D}$ of the complex plane, referred to as *LMI regions*, are defined as:

$$\mathscr{D} = \{z \in \mathbb{C} : f_{\mathscr{D}}(z) < 0\} \tag{23}$$

where $f_{\mathscr{D}}$ is the *characteristic function*, defined as:

$$f_{\mathscr{D}}(z) = \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + \beta_{kl} z + \beta_{lk}\bar{z}]_{k,l\in[1,m]} \tag{24}$$

where $\alpha = \alpha^T \in \mathbb{R}^{m\times m}$ and $\beta \in \mathbb{R}^{m\times m}$. Hence, the TS error system (16), with the error-feedback control law (21), has its poles in $\mathscr{D}$ if there exist $X_{\mathscr{D}} = X_{\mathscr{D}}^T > 0$ and matrices $K_i$ such that:

$$\left[\alpha_{kl} X_{\mathscr{D}} + \beta_{kl}\left(A + B_j K_i\right) X_{\mathscr{D}} + \beta_{lk} X_{\mathscr{D}}\left(A + B_j K_i\right)^T\right]_{k,l\in[1,m]} < 0 \qquad i,j = 1,\dots,N \tag{25}$$

Two issues arising when using (22) and (25) for design are that:

- conditions (22) and (25) are Bilinear Matrix Inequalities (BMIs), since the products of the variables $K_i$ by the matrices $X_s$ and $X_{\mathscr{D}}$ appear. In order to reduce the BMIs to Linear Matrix Inequalities (LMIs), a common Lyapunov matrix $X_s = X_{\mathscr{D}} = X$ is chosen, and the change of variables $\Gamma_i \triangleq K_i X$ is introduced;
- in TS systems where only the input matrix $B$ changes according to the considered subsystem, the solution provided by (22) and (25) exhibits too conservatism, due to the fact that a given $K_i$ has to guarantee stability/pole clustering for all the possible $B_j$. In order to reduce such conservatism, a gridding approach is considered for obtaining the TS model, and the design conditions are written at the grid points only. Even though the stability and the pole clustering specification are theoretically guaranteed only at the grid points, from a practical point of view such specifications should be guaranteed by choosing a grid of points dense enough.

Hence, the conditions to be used for finding the gains $K_i$ are the following:

$$\begin{pmatrix} -X & AX + B_i\Gamma_i \\ XA^T + \Gamma_i^T B_i^T & -X \end{pmatrix} < 0 \qquad i = 1,\dots,N \tag{26}$$

$$\left[\alpha_{kl} X + \beta_{kl}\left(AX + B_i\Gamma_i\right) + \beta_{lk}\left(XA^T + \Gamma_i^T B_i^T\right)\right]_{k,l\in[1,m]} < 0 \qquad i = 1,\dots,N \tag{27}$$

These LMIs can be solved efficiently using available software, e.g. the YALMIP toolbox [15] with SeDuMi solver [16].

## 4 Simulation Results

The TS control design technique described in Section 3 has been applied to the error model of the PEM Fuel Cell presented in Section 2, where the state matrix is given as follows:

$$A = \begin{pmatrix} -21.8644 & 0 & 1.9881\cdot 10^{8} \\ 0 & -37.4749 & 3.4076\cdot 10^{8} \\ 3.6290\cdot 10^{-6} & 2.1770\cdot 10^{-6} & -52.7940 \end{pmatrix}$$

By considering that the supply and the return manifold pressures $p_{rm}$ and $p_{sm}$ can take values in given intervals:

$$p_{rm} \in \left[1.3\cdot 10^5, 13\cdot 10^5\right] \qquad p_{sm} \in \left[1.3\cdot 10^5, 13\cdot 10^5\right]$$

it is obtained that the elements of the input matrix vary in the indicated ranges:

$$B = \begin{pmatrix} b_{11}(p_{sm}) & 0 \\ 0 & b_{22}(p_{rm}) \\ 0 & 0 \end{pmatrix} \in \begin{pmatrix} \left[1.3 \cdot 10^5, 13 \cdot 10^5\right] & 0 \\ 0 & \left[1.3 \cdot 10^5, 13 \cdot 10^5\right] \\ 0 & 0 \end{pmatrix}$$

The non-linear sector approach has been applied to obtain a TS model by dividing these intervals into 30 points each. This leads to a grid of 900 pairs $(b_{11}, b_{22})$, i.e., submodels.

Using this model, a TS controller with the structure (20) has been designed using (26) to assure stability and using (27) to achieve pole clustering in a circle of radius 0.4 and center $(0.599, 0)$. This controller needs an observer to estimate the error between the reference and the real states of the PEM fuel cell. Even though the observer is implemented as a TS observer (see [10] for more details about TS observers) due to the variability of the values $b_{11}(p_{sm})$ and $b_{22}(p_{rm})$, the design of the observer can be performed using an LTI exact pole placement technique, since both the $A$ and the $C$ matrices of the TS PEM Fuel Cell error model are constant. In particular, in this work the error observer eigenvalues have been put in $\{0.3, 0.25, 0.2\}$.

The results shown in this paper refer to a simulation which lasts $300\,s$, where abrupt change in the stack current $I_{st}(t)$ and the desired oxygen excess ratio $\lambda_{O2}^{ref}(t)$ were introduced. The PEM fuel cell initial states have been chosen as follows:

$$\begin{bmatrix} p_{sm}(0) \\ p_{rm}(0) \\ m_{H2,an}(0) \\ m_{O2,ca}(0) \end{bmatrix} = \begin{bmatrix} 1.6 \cdot 10^5\,Pa \\ 1.6 \cdot 10^5\,Pa \\ 5 \cdot 10^{-4}\,kg \\ 0.01\,kg \end{bmatrix}$$

A Gaussian noise with zero mean and standard deviation equal to 5 ‰ of the measurement has been considered for both the available sensors (state variables $p_{sm}$ and $p_{rm}$). Fig. 1 shows the evolution of the stack current $I_{st}$ during the simulation and the tracking of the desired oxygen excess ratio. It can be seen that the reference is correctly followed independently of the values taken by the stack current. This is done by changing the compressor mass flow $W_{cp}$ and the return manifold outlet constant $k_{rm,out}$, taking into account both the feedforward and the feedback control law, as shown in Fig. 2. Finally, the values taken by the state variables are shown in Fig. 3.

## 5 Conclusions

In this paper, the problem of controlling the oxygen stoichiometry for a PEM fuel cell has been solved. The proposed solution relies on the use of a reference model that describes the desired behaviour. The resulting nonlinear error model is brought to a TS form that is used for designing a TS controller using LMI-based techniques. The results obtained in simulation environment have demonstrated the effectiveness of the proposed technique. As future work, the proposed TS approach will be compared with the LPV approach in order to see the benefits and drawbacks of each one of these two techniques.

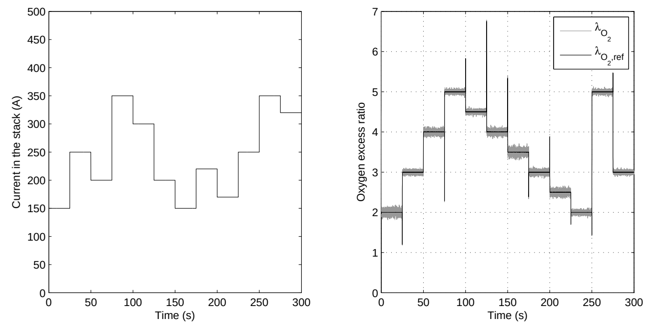


**Fig. 1.** Current in the stack and oxygen excess ratio.

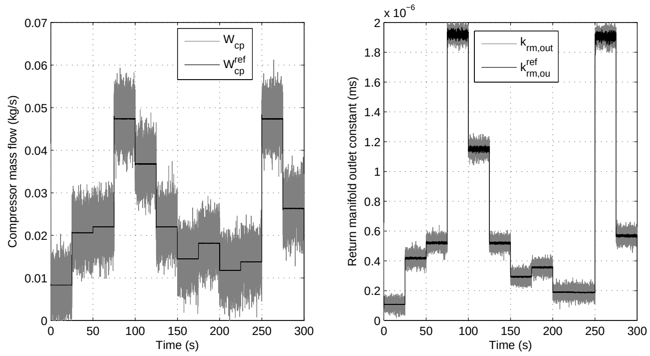


**Fig. 2.** Input variables.

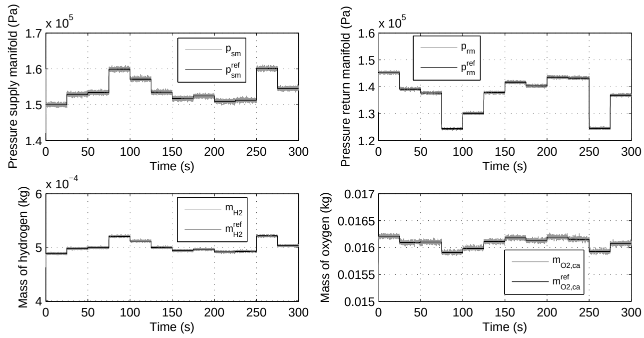


**Fig. 3.** State variables.

## Acknowledgement

This work has been funded by the Spanish MINECO through the project CYCYT SHERECS (ref. DPI2011-26243), by the European Commission through contract i-Sense (ref. FP7-ICT-2009-6-270428) and by AGAUR through the contract FI-DGR 2013 (ref. 2013FIB00218).

## References

1. J. T. Pukrushpan, A. G. Stefanopoulou, and H. Peng, *Control of Fuel Cell Power Systems: Principles, Modeling, Analysis and Feedback Design*. Series Advances in Industrial Control, Springer, 2004.
2. A. Niknezhadi, M. Allué, C. Kunusch, and C. Ocampo-Martínez, "Design and implementation of LQR/LQG strategies for oxygen stoichiometry control in PEM fuel cells based systems," *Journal of Power Sources*, vol. 196, no. 9, pp. 4277–4282, 2011.
3. C. Ziogou, S. Papadopoulou, M. C. Georgiadis, and S. Voutetakis, "On-line nonlinear model predictive control of a PEM fuel cell system," *Journal of Process Control*, vol. 23, no. 4, pp. 483 – 492, 2013.
4. J. T. Pukrushpan, H. Peng, and A. G. Stefanopoulou, "Control-oriented modeling and analysis for automotive fuel cell systems," *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 126, no. 1, pp. 14–25, 2004.
5. F. D. Bianchi, C. Kunusch, , and C. O. amd R. S. Sánchez-Peña, "A gain-scheduled LPV control for oxygen stoichiometry regulation in PEM fuel cell systems," *IEEE Transactions on Control Systems Techonology*, vol. (in preview), 2014.
6. S. de Lira, V. Puig, J. Quevedo, and A. Husar, "LPV observer design for PEM fuel cell system: Application to fault detection," *Journal of Power Sources*, vol. 196, no. 9, pp. 4298 – 4305, 2011.
7. T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-15, no. 1, pp. 116–132, 1985.
8. S. Olteanu, A. Aitouche, M. Oueidat, and A. Jouni, "PEM fuel cell modeling and simulation via the Takagi-Sugeno Fuzzy model," in *International Conference on Renewable Energies for Developing Countries (REDEC)*, 2012, pp. 1–7.
9. K. Tanaka and H. O. Wang, *Fuzzy control systems design and analysis: A linear matrix inequality approach*. John Wiley and Sons, Inc., 2001.
10. P. Korba, R. Babuška, H. Verbruggen, and P. Frank, "Fuzzy gain scheduling: controller and observer design based on Lyapunov method and convex optimization," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 3, pp. 285–298, 2003.
11. C. Kunusch, P. Puleston, and M. Mayosky, *Sliding-Mode control of PEM fuel cells*. London, U.K.: Springer-Verlag, 2012.
12. A. Aitouche, Q. Yang, and B. Ould Bouamama, "Fault detection and isolation of PEM fuel cell system based on nonlinear analytical redundancy," *The European Physical Journal Applied Physics*, vol. 54, no. 2, 2011.
13. H. Wang, K. Tanaka, and M. Griffin, "An approach to fuzzy control of nonlinear systems: stability and design issues," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 1, pp. 14–23, 1996.
14. M. Chilali and P. Gahinet, "$H_\infty$ Design with Pole Placement Constraints: An LMI Approach," *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 358–367, 1996.
15. J. Löfberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *Proceedings of the CACSD Conference*, 2004.
16. J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optimization methods and software*, vol. 11-12, pp. 625–653, 1999.